The Sensitivity of a Method to Predict a Capacitor's Frequency Characteristic

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***Abstract*—A joint effort between the U.S. Naval Academy and the National Institute of Standard and Technology (NIST) resulted in the development of a method to characterize the capacitance and dissipation factor of a set of commercial standard four terminal-pair (4TP) capacitors. The method depends on network analyzer impedance measurements at high frequencies (40 MH–200 MHz) and a regression of these measurements down to the frequency range of 10 MHz–1 kHz. This paper provides an analysis of the sensitivity of the regression parameters and the high-frequency impedance measurements.**

***Index Terms*—Capacitance, dissipation factor, four terminal-pair capacitor, high frequency, impedance, precision measurements, sensitivity analysis.**

I. Introduction

SEVERAL groups have worked on the characterization of four terminal-pair (4TP) capacitance standards at high frequencies [1]–[9]. The work originally started with Cutkosky and Jones of the National Institute of Standards and Technology (NIST) and was followed by Suzuki, Aoki, Yokoi, Yonekura, and Wakasugi of HP Japan. This paper describes a variation of the technique described by Aoki *et al.* to predict a capacitor's frequency characteristic [1]–[3] The method is sensitive to regression parameter selection and the paper gives a detailed analysis of the techniques used to calculate reasonable values for these parameters.

The principles of a capacitor frequency characteristic prediction (CFCP) method is described in [1] and some practical aspects are addressed in [3] and [9]. As a background introduction to our discussion, the practical realization of the CFCP method is summarized.

II. Practical Realization of the CFCP Method

The following equation represents the calculation of a 4TP impedance from driving-point impedances (measurable with a network analyzer) [1], [3] (see Fig. 1):

$$Z_{4\mathrm{tp}} = \sqrt{\frac{Z_{22}}{Z_{11}-Z_{11s3}}}\Big(\sqrt{(Z_{11}-Z_{11s2})(Z_{44}-Z_{44s3})} - \sqrt{(Z_{11}-Z_{11s3})(Z_{44}-Z_{44s2})}\Big). \quad (1)$$

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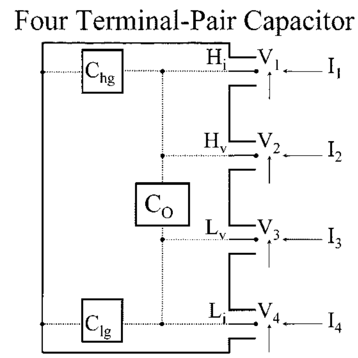

Fig. 1. Four terminal-pair capacitance standard.

Z_{ii} is the driving-point impedance at port i with all other ports left open, Z_{iisj} is the driving-point impedance at port i with port j shorted and all other ports left open.

Present measurement requirements at NIST have made it necessary to develop an updated approach for determining four terminal-pair capacitance and dissipation factor in the frequency range of 1 kHz–10 MHz. In the proposed procedure, a network analyzer is used to measure the driving point impedance values at very high frequencies and a regression analysis is performed to predict the capacitor's frequency characteristic at frequencies from 1 kHz to 10 MHz.

In order to measure the necessary impedances, namely, Z_{ii} and Z_{iisj}, the following procedure is performed.

Step 1) The network analyzer is verified and calibrated over the frequency ranges used to conduct the driving-point impedance measurements.

Step 2) Impedances Z_{ii} and Z_{iisj} are measured using the network analyzer over the specific frequency ranges. Equation (1) demonstrates the need to know the values of impedance differences: $Z_{11}-Z_{11s2}$, $Z_{11}-Z_{11s3}$, $Z_{44}-Z_{44s2}$, and $Z_{44}-Z_{44s3}$. Throughout the rest of this paper $Z_{ii}-Z_{iisj}$ is used to represent these difference measurements.

Step 3) A circuit diagram [4], [5] of a 4TP capacitance standard is shown in Fig. 1. It is assumed that the capacitive component of this impedance does not change with frequency. Capacitances C_0, C_{hg}, and C_{lg} (see Fig. 1) are measured at 1 kHz using a high-accuracy capacitance bridge. Capacitance C_0 is the nominal capacitance measured between the high and low terminals, C_{hg} is the capacitance measured between the high and ground terminals, and C_{lg} is the capacitance measured between the low and ground.

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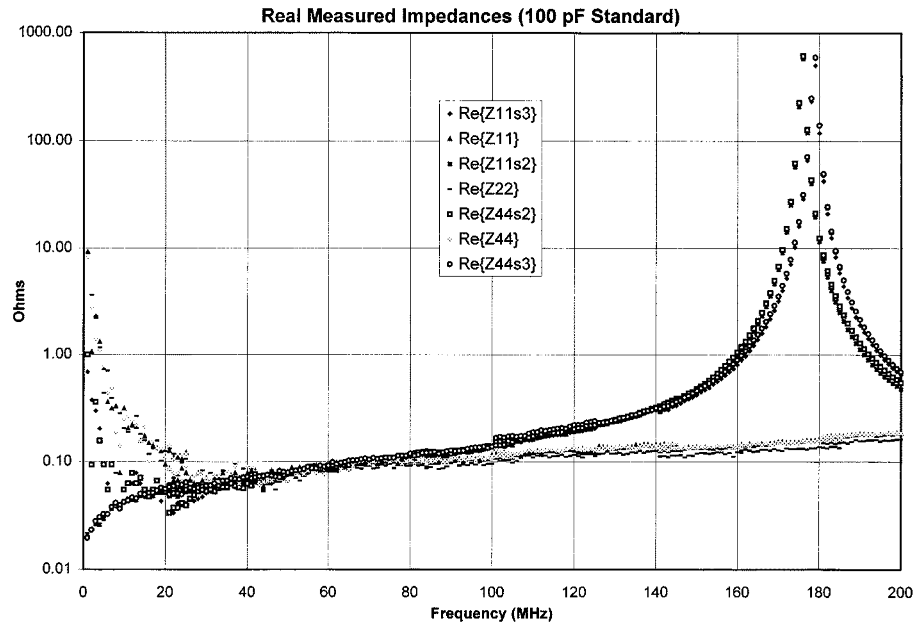

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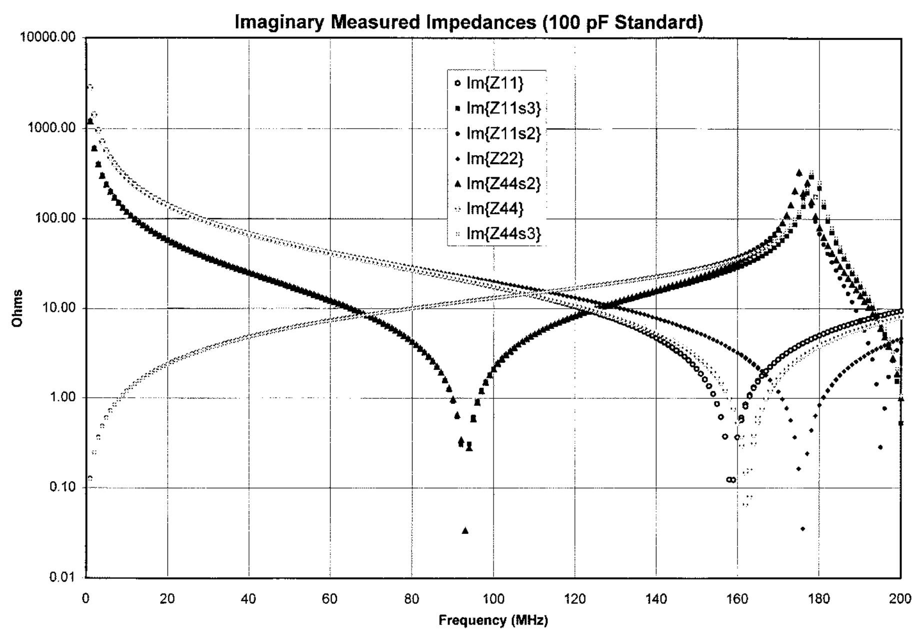


(b)

Fig. 2. Real and imaginary parts of the measured impedance for a 100 pF capacitor.

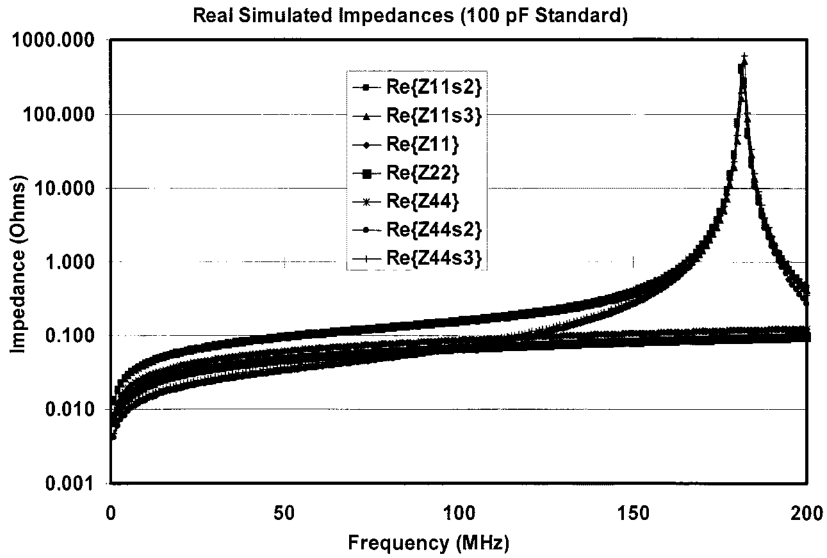
Step 4) It is assumed that the change in the capacitor's frequency characteristic is due only to the inductive and resistive impedance components. For example, impedance $Z_{ii-iisj}$ is represented in the form

$$Z_{ii-iisj} = R_{ii-iisj} + j\left(2\pi f L_{ii-iisj} - \frac{1}{2\pi f C_{ii-iisj}}\right). \quad (2)$$

In the above equation, circuit analysis techniques are used to solve the equivalent capacitance at each of the ports on a capacitor, C_{ii} and C_{iisj}. The remaining resistive and inductive components are solved using regression techniques. To reduce the influence of skin effect, the measured real component must first be normalized using the formula

$$R_{\mathrm{Normalized}} = \frac{R_{\mathrm{Measured}}}{\sqrt{f_{\mathrm{Measured}}}}. \quad (3)$$

Step 5) This approach simplifies the regression process so that resistive and inductive components may be analyzed separately, i.e., the resistive components are calculated from the real part of the network analyzer measurements after being normalized using



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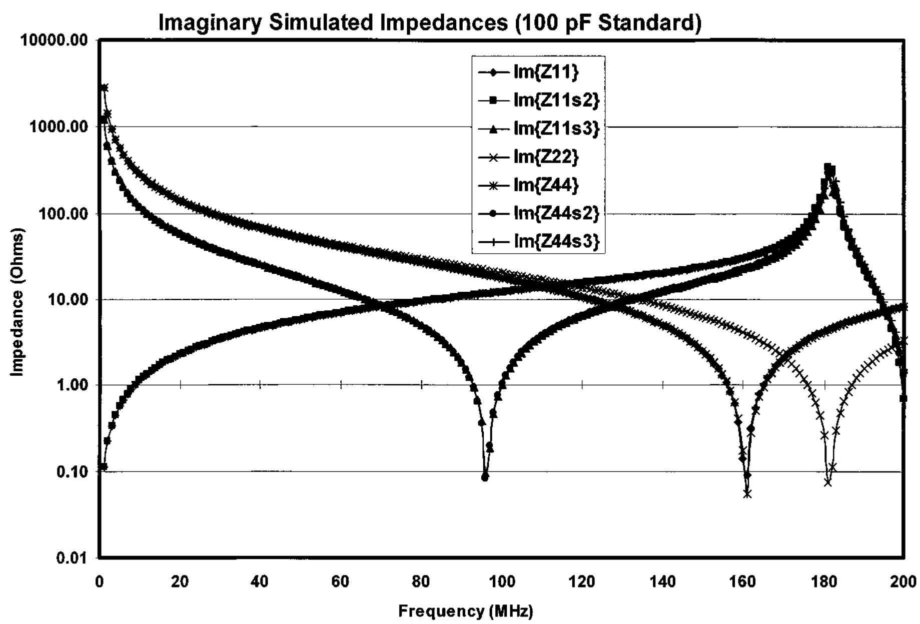


(b)

Fig. 3. Simulated values for impedance using a model for a 100 pF capacitor.

(3). Likewise, the inductive components are calculated from the imaginary part of the network analyzer measurements according to (2). The form of the regression used is

$$A(f) = A_0 + f^P A_r \tag{4}$$

where

$$A_0 = \mathrm{mean}(L_{ii-iisj}) - A_r \mathrm{mean}(f) \tag{5}$$

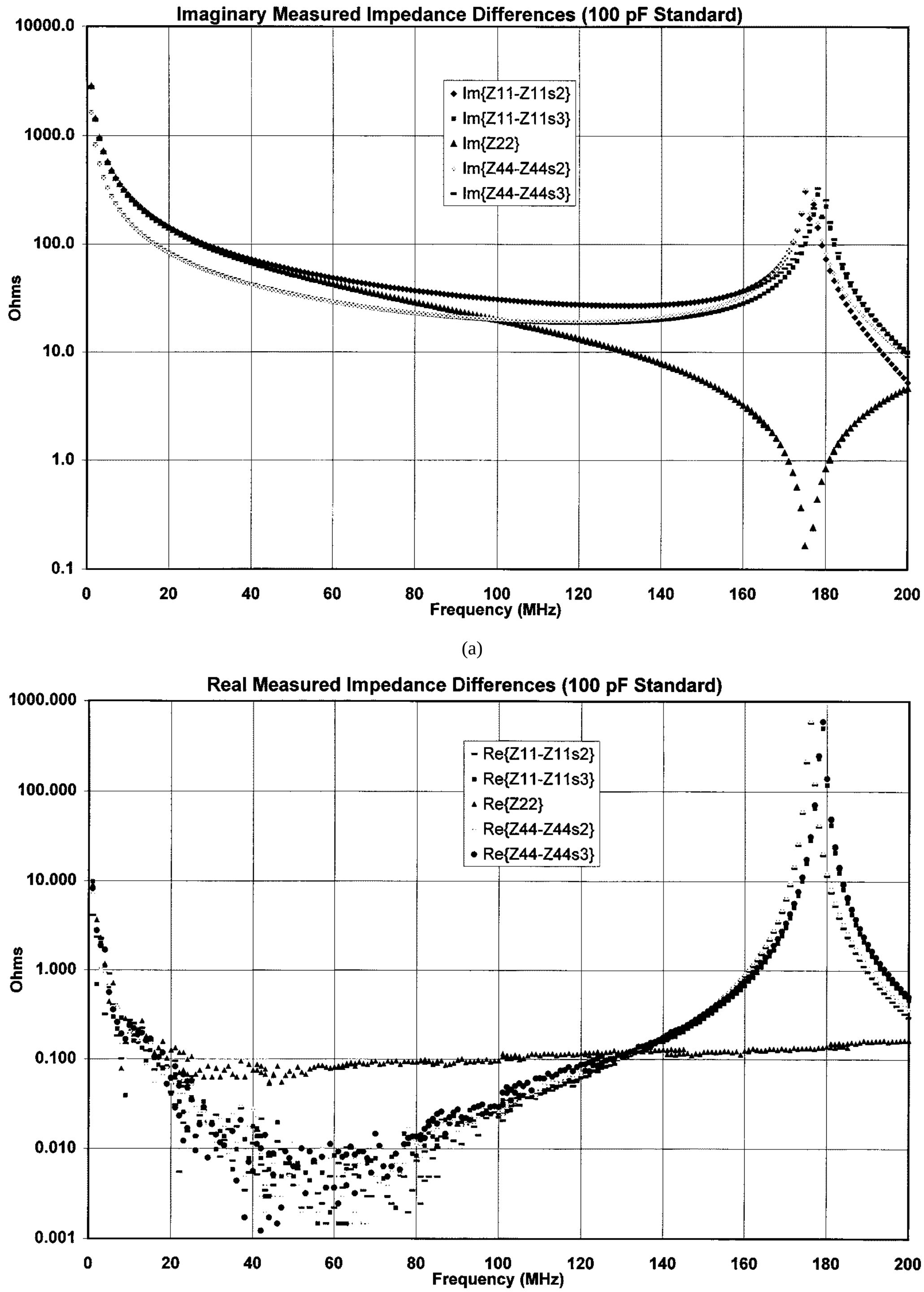
$$A_r = \frac{\sum[L_{ii\text{–}iisj_n} - \mathrm{mean}(L_{ii\text{–}iisj})] \sum[f_n - \mathrm{mean}(f)]}{\sum[L_{ii\text{–}iisj_n} - \mathrm{mean}(L_{ii\text{–}iisj})]^2}. \tag{6}$$

n is the number of measurements taken using the network analyzer, and p is the frequency exponent. A more detailed explanation on the selection of regression parameters, as well as measurement frequency ranges, is given below.

Step 6) The last step is to predict the value of each driving-point impedance at a desired frequency. The regression (4) may be used to predict $R_{ii-iisj}$ and $L_{ii-iisj}$ for any frequency in the range 1 kHz–10 MHz. Equations (1) and (2) are then used to predict the resulting impedance, $Z_{4\mathrm{TP}}$.

III. Regression Parameter Selection

By using a network analyzer to measure the capacitor's impedances, several limitations in selecting the measurement frequency ranges take place.



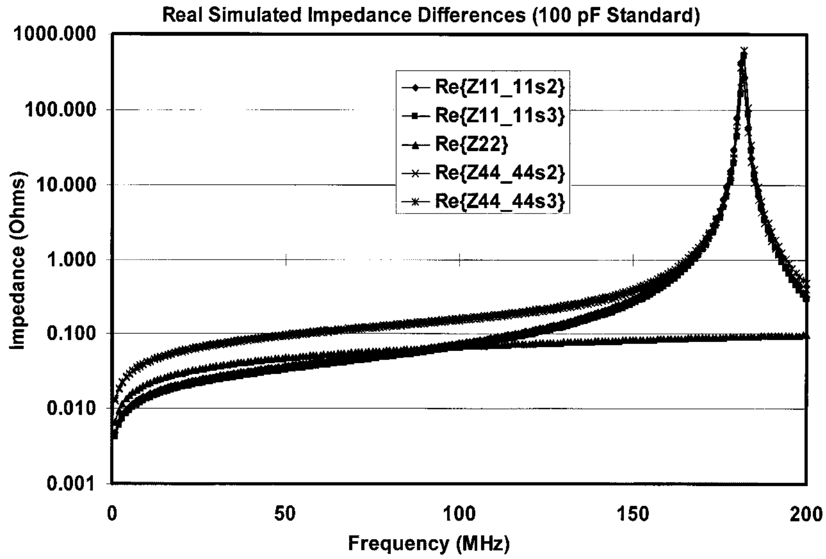
(a)

(b)

Fig. 4. The 100 pF capacitor impedance measurements to be compared with simulations in Fig. 3.

Network analyzers are calibrated using standard open, short, and load devices. The standard load is a calibrated 50 Ω resistor. In order to maintain high accuracy, the network analyzer should measure the test capacitor at frequencies where its impedance is on the order of 50 Ω. To further reduce the influence of any network analyzer offset errors, Z_{11} and Z_{44} are measured twice. For example, Z_{11} is measured just prior to measuring Z_{11s2} and also just before measuring Z_{11s3}.

4TP air dielectric capacitors exhibit extremely low dissipation factor behavior (on the order of 10^{-5} or lower). In general, network analyzers cannot resolve this behavior. To overcome this limitation, the measurements may be made at extremely high frequencies where the dissipation factor components dominate the behavior. However, these are the same frequencies where the real components of the measurements are severely influenced by skin effect. A tradeoff must be made.



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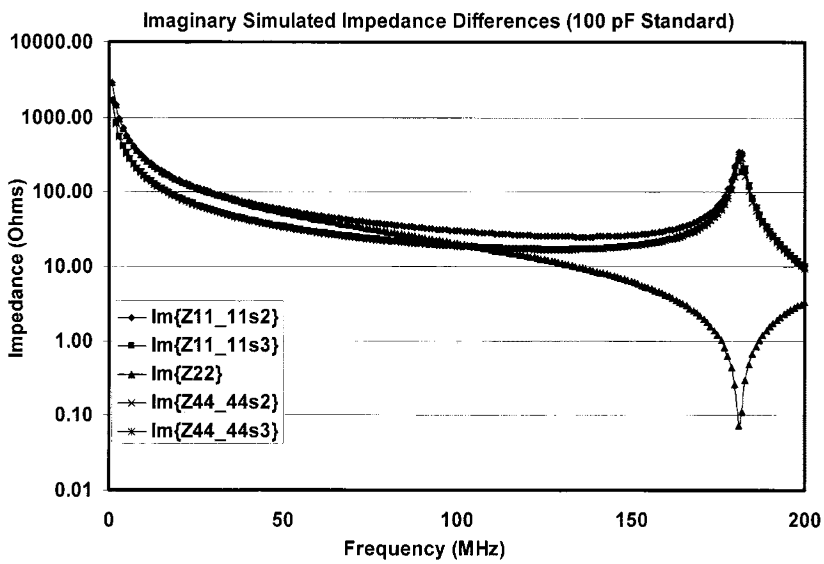

(d)

Fig. 4. (*Continued*.) The 100 pF capacitor impedance measurements to be compared with simulations in Fig. 3.

The third guideline in the selection of measurement frequencies is related to the frequency characteristic of each port measured. Since the regression is used to estimate the capacitor's frequency characteristic, the measurements must be made far away from any resonances (frequencies where the impedance is nonreactive).

The complexity of the problem lies in the fact that there are seven different impedances to be measured in order to estimate the capacitor's frequency characteristic. The real and imaginary parts of the impedance for a number of air capacitors were measured in the range 20 MHz–200 MHz using a network analyzer. The example in Fig. 2 shows impedance measured for a 100 pF air capacitor. The results show the significant difference in measured values between the real and the imaginary parts of the impedances. This figure and subsequent figures are given to demonstrate the complexity of the optimization process, not necessarily the details. To achieve optimal network analyzer measurements that are around 50 O, the frequency ranges have to be around 100 MHz (note $\log(50) \sim 1.7$). Note also the number of resonant frequencies to be avoided. Air capacitors have very small dissipation factor and the measured real component is very small. At lower frequencies the imaginary component of a capacitor (and thus, Q factor) is sufficiently high that the network analyzer is unable to resolve the real component.

Directly related to the frequency range selection is the selection of the regression exponent parameter, p. A detailed model of a capacitor is necessary in order to find the best regression fit. Simulations were performed based on the model published by Yonekura and Wakasugi [3]. Using nominal values for resistive, inductive, and capacitive components of the model capacitor, a procedure was developed to simulate a wide range of network analyzer measurements. The model calculations were then compared to the measurements of a real capacitor. The measure-

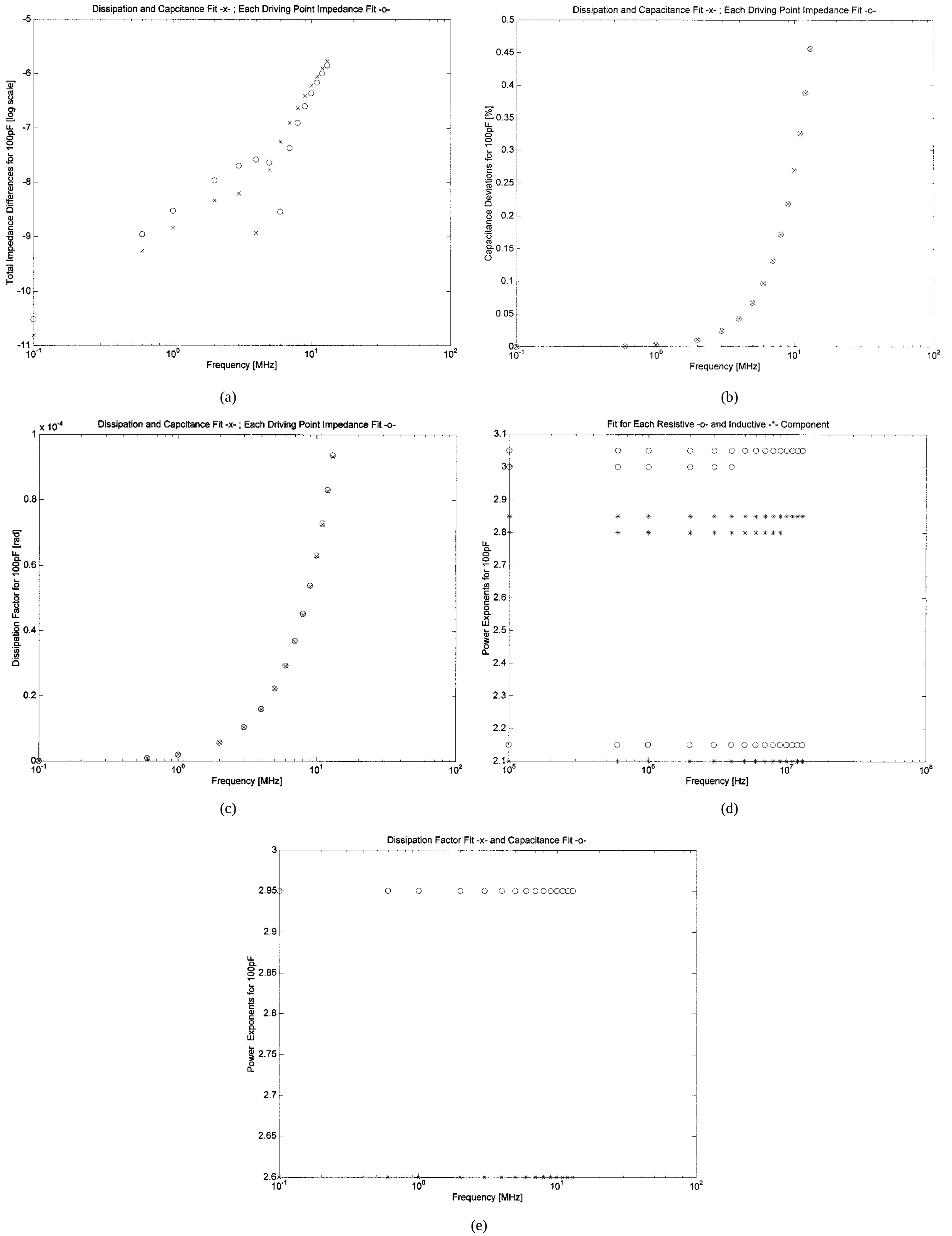

Fig. 5. (a) Total impedance differences for two different sets of regression parameters. (b) Capacitance characteristics for two different sets of regression parameters. (c) Dissipation factor results for two different sets of regression parameters. (d) Power exponents for resistive and inductive regressions parameters obtained by fitting each driving-point impedance curve. (e) Power exponents for resistive and inductive regressions parameters obtained by fitting dissipation factor curve and capacitance differences curve separately.

ment vs. simulation data show good agreement (see Fig. 3 for the simulations and Fig. 4 for the measurements). Based on this agreement, it was concluded that it is acceptable to use the given model as a starting point in a search for an optimal set of regression parameters.

The regression method used with a varied set of regression parameters was applied to the simulated data to predict the capacitor's frequency characteristic. Since the impedance of the model capacitor could be calculated at any frequency, the criterion for optimal parameter selection was based on the best fit between the predictions using the regression method and impedance calculations using the model.

A search program was developed to test a range of regression parameter values using small incremental steps. Since the goal of this method is to accurately predict a capacitor's dissipation factor and capacitance change over a range of frequencies, these two properties were optimized independently and sets of regression parameters for resistive and inductive components were found. One way to set a selection criterion is, for each of the 4TP air capacitors evaluated in the study, to determine a single exponent parameter for use in the inductance regressions and a single exponent parameter for use in the resistance regressions. Another way to set a selection criterion is to search for the best fit for each driving-point impedance. These impedances are given in (1). In this way ten different values (five for the resistive and five for the inductive components) for the frequency exponent parameter, p, were found for each 4TP capacitor. Fig. 5(a)–(e) shows several plots comparing results from two different sets of parameter values. Fig.5(a) compares total impedance results, showing better results when each driving point impedance curve is addressed separately compared to the cumulative parameter for the dissipation and capacitance difference result. Fig. 5(b) and (c) compare capacitance characteristics and dissipation factor results. These results are similar for both selection procedures. Fig. 5(d) and (e) list power exponents for the resistive and inductive regressions obtained using the presented search procedure.

The search program was also tested using different network analyzer measurement frequency ranges. It was found that the values for the frequency exponent parameter p change significantly from range to range. This leads to the conclusion that the CFCP method can not be implemented successfully without having a detailed model of a test capacitor. Future research should address the results of simulations on the sensitivity of the CFCP method due to variations in the regression parameters.

IV. Conclusion

The results of the analysis of the CFCP method's sensitivity to exponent parameter variation have shown that this sensitivity is a significant contributor in the uncertainty analysis of NIST's capacitance standard measurement system. A detailed description and uncertainty analysis of this system are addressed in another paper.

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